Finite Math - Spring 2017

Lecture Notes - 4/28/2017

Homework

- Section 5.3 - 1, 2, 5, 6, 9, 17, 19, 21, 23, 25, 27, 35, 43, 50, 53

Section 5.3 - Linear Programming in Two Dimensions: A Geometric Approach

General Description of Linear Programming. In a *linear programming problem*, we are concerned with *optimizing* (finding the maximum and minimum values, called the *optimal values*) of a linear *objective function* z of the form

$$z = ax + by$$

where a and b are not both zero and the *decision variables* x and y are subject to *constraints* given by linear inequalities. Additionally, x and y must be nonnegative, i.e., $x \geq 0$ and $y \geq 0$.

The following theorems give us information about the solvability and solution of a linear programming problem:

Theorem 1 (Fundamental Theorem of Linear Programming)**.** *If the optimal value of the objective function in a linear programming problem exists, then that value must occur at one or more of the corner points of the feasible region.*

Theorem 2 (Existence of Optimal Solutions)**.**

(A) *If the feasible region for a linear programming problem is bounded, then both the maximum value and the minimum value of the objective function always exist.*

(B) *If the feasible region is unbounded and the coefficients of the objective function are positive, then the minimum value of the objective function exists, but the maximum value does not.*

(C) *If the feasible region is empty, then both the maximum value and the minimum value of the objective function do not exist.*

Geometric Method for Solving Linear Programming Problems.

Procedure (Geometric Method for Solving a Linear Programming Problem with Two Decision Variables)**.**

(1) *Graph the feasible region. Then, if an optimal solution exists according to Theorem 2, find the coordinates of each corner point.*

(2) *Construct a* corner point table *listing the value of the objective function at each corner point.*

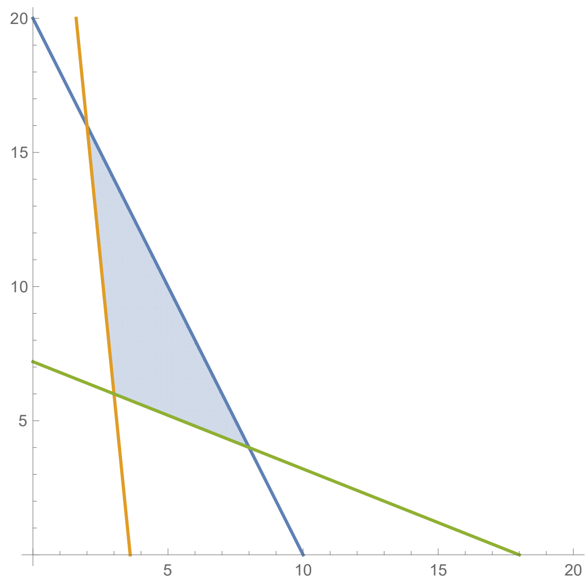
(3) *Determine the optimal solution(s) from the table in Step 2 (smallest=minimum, largest=maximum).*

(4) *For an applied problem, interpret the optimal solution(s) in terms of the original problem.*

Example 1. *Maximize and minimize* $z = 3x + y$ *subject to the inequalities*

$$\begin{aligned} 2x + y &\leq 20 \\ 10x + y &\geq 36 \\ 2x + 5y &\geq 36 \\ x, y &\geq 0 \end{aligned}$$

Solution. *We begin by graphing the feasible region*



Since this is a bounded region, by part (A) of Theorem 2, this problem has both a maximum and a minimum value. Now we find the corner points, of which there are three.

colors	*system*	*corner point*
blue and orange	$\begin{cases} 2x + y = 20 \\ 10x + y = 36 \end{cases}$	$(2, 16)$
blue and green	$\begin{cases} 2x + y = 20 \\ 2x + 5y = 36 \end{cases}$	$(8, 4)$
orange and green	$\begin{cases} 10x + y = 36 \\ 2x + 5y = 36 \end{cases}$	$(3, 6)$

Now, we check the value of z *at these corner points:*

corner point	$z = 3x + y$
$(2, 16)$	$z = 3(2) + (16) = 6 + 16 = 22$
$(8, 4)$	$z = 3(8) + (4) = 24 + 4 = 28$
$(3, 6)$	$z = 3(3) + (6) = 9 + 6 = 15$

Then, we see that the minimum value is 15 *and occurs at* $(3, 6)$ *and the maximum value is* 28 *and occurs at* $(8, 4)$*.*

Example 2. *Maximize and minimize* $z = 2x + 3y$ *subject to*

$$\begin{aligned} 2x + y &\geq 10 \\ x + 2y &\geq 8 \\ x, y &\geq 0 \end{aligned}$$

Solution. *Minimum of* $z = 14$ *at* $(4, 2)$*. No maximum.*

Example 3. *Maximize and minimize* $P = 30x + 10y$ *subject to*

$$\begin{aligned} 2x + 2y &\geq 4 \\ 6x + 4y &\leq 36 \\ 2x + y &\leq 10 \\ x, y &\geq 0 \end{aligned}$$

Solution. *Minimum of* $P = 20$ *at* $(0, 2)$*. Maximum of* $P = 150$ *at* $(5, 0)$*.*

Example 4. *Maximize and minimize* $P = 3x + 5y$ *subject to*

$$\begin{aligned} x + 2y &\leq 6 \\ x + y &\leq 4 \\ 2x + 3y &\geq 12 \\ x, y &\geq 0 \end{aligned}$$

Solution. *No optimal solutions.*